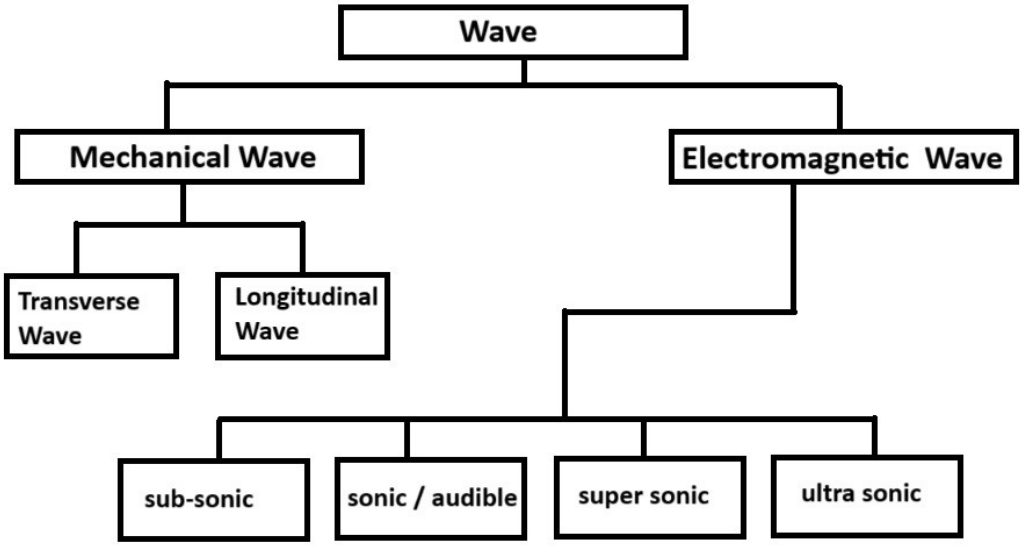
6. Superposition Of Waves



Characteristics of ware Motion :-

1. Wave Motion is not the motion of the particles but it is the motion of an oscillatory disturbance.
2. In wave motion energy is transferred from particle to article of the medium.
3. All the particles of medium perform S.H.M about their mean position.
4. For the propagation of wave, medium is necessary. It should be elastic & possesses inertia.
5. The wave motion is periodic in time as well as periodic in space.

Question :- Explain why the wave motion is doubly periodic phenomenon ?

When the waves are traveling through medium, we observe that at any point of the medium, the form of wave repeats after equal interval of time. This shows that the wave motion is periodic in time.

Also at any instant, the form of wave repeats at equal distances, this shows that the wave motion is periodic in space.

Thus, the wave motion is a doubly periodic phenomenon or it has double periodicity.

Simple Harmonic Progressive wave (S.H.P wave) :-

Define : The waves which Contineously travels in a same direction, are called the progressive wave. When these waves travel through a medium, the particles of medium perform vibration about their mean position . It these vibrations are simple harmonic, these waves are called as S.H.P waves.

Characteristics of SHP Waves :-

i. In progressive wave net transfer of energy takes place.
ii. In progressive wave all the particles of medium vibrate about their mean position performing S.H.M.
iii. In progressive wave amplitude, frequency of all the particles in medium is constant & the same.
iv. In progressive wave the particles of medium vibrate with different in phase.

Amplitude of Wave (a) :-

Define : The maximum displacement of a particle from its mean position Is called amplitude of wave.

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Period of Wave (T) :-

Define : The time taken by a particle to complete one vibration, is called period of wave.

Frequency of Wave (n) :-

Define : The number of vibrations performed by a particle in one second, is called frequency of wave.

Wavelength of wave (λ) :-

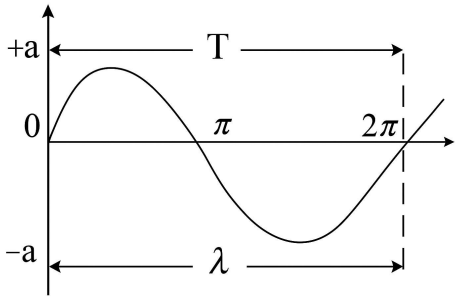
Define : The distance between two successive particles which are in same state of vibration (i.e. differ in phase by 2π) is called wavelength of wave.

Velocity of wave (V) :-

Define : The distance travelled by an Oscillatory disturbance per unit time, is called velocity of wave.

Relation between velocity, Frequency & Wavelength of Wave :-

Consider a SHP wave, which covers a distance, equal to one wavelength (λ) in one period (T) Then the velocity of wave is given by;



$$Velocity = \frac{Displacement}{Time}$$

$$V = \frac{\lambda}{T}$$

$$V = \frac{1}{T} \times \lambda$$

$$V = n\lambda \qquad | \because \; n = \frac{1}{T} \; |$$

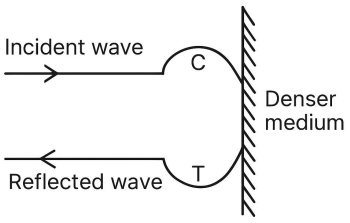
Classification of Waves :-

The waves can be classified as, electromagnetic waves & mechanical waves.
The waves which do not necessary the material medium for their propagation, is known as electromagnetic waves. Ex. Light waves, radiations etc.
The waves which require material medium for their propagation, are called as mechanical waves. Ex. Sound wave, waves along vibrating string.
The mechanical waves further classified as,

1) Transverse waves (particles perpendicular to direction of propagation of wave)
2) Longitudinal waves (particles parallel to direction of propagation of wave)

Reflection of Waves & change of phase :-

1. **Reflection of transverse wave :-**
   a) From Denser medium (Rigid wall)
   When the crest of transverse wave is incident on denser medium, it gets reflected. Therefore, here the wave velocity & particle velocity both are reversed. Hence crest is reflected as through & vice-versa. Therefore, there is a phase change of π^c between incident & reflected wave.



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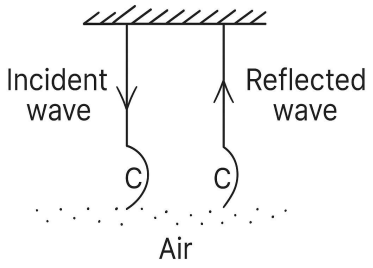
b) From Rarer medium (air)
When transverse wave incident on rarer medium, then wave velocity reversed & particle velocity do not change, because the particles of rarer medium are free to move. Therefore, there is no phase change due to reflection. So, crest is reflected as crest & trough is reflected as trough.



2. <u>Reflection of longitudinal wave (sound wave). :-</u>

a) From denser wave :-
When compression of longitudinal wave incident on denser medium . both wave velocity & particle velo reversed & again are in same direction. Therefore, compression is reflected as compression & rarefaction is reflected as rarefaction. Thus, there is a phase change of TT radian between incident & reflected wave.

b) From rarer medium :-
When longitudinal wave is incident on rarer medium wave velocity reversed but particle velocity do not change because the particle of rarer medium is free to move Thus there is no phase change dues to reflection. Hence compression reflected as rarefaction & vice versa.

Amplitude Of the Resultant Wave Produced Due To Superposition of Two Waves:

Consider two waves having the same frequency but different amplitude A_1 and A_2. Let these waves differ
in phase by ϕ . The displacement of each wave at x = 0 is given as

$y_1 = A_1 \sin \omega t$ .....(1)

$y_2 = A_2 \sin(\omega t + \phi)$ .....(2)

According to principle of superposition of wave ;

$y = y_1 + y_2$

$y = A_1 \sin \omega t + A_2 \sin(\omega t + \phi)$

$y = A_1 \sin \omega t + A_2 \sin \omega t \,.\cos \phi + A_2 \cos \omega t \,.\sin \phi$

$y = (A_1 + A_2.\cos \phi) \sin \omega t + A_2 \cos \omega t \,.\sin \phi$

If we write,

$A_1 + A_2.\cos \phi = A \cos \phi$ ..... (3)

And $A_2.\sin \phi = A.\sin \phi$ .....(4)

We get,

$y = A \cos \phi \,.\sin \omega t + A \sin \phi \,.\cos \omega t$

$y = A \sin(\omega t + \phi)$ .....(5)

This is the equation of the resultant wave. It has the same frequency as that of the interfering waves.

Squaring and adding equation (3) & (4)

$A^2 \cos^2 \phi + A^2 \sin^2 \phi = (A_1 + A_2.\cos \phi)^2 + {A_2}^2 \,.\sin^2 \phi$

$A^2(1) = [\,{A_1}^2 + 2A_1A_2 \cos \phi + {A_2}^2 \cos^2 \phi\,] + {A_2}^2 \,.\sin^2 \phi$

$A^2 = {A_1}^2 + 2A_1A_2 \cos \phi + {A_2}^2(1)$ $\quad | \because {A_2}^2 \cos^2 \phi + {A_2}^2 \,.\sin^2 \phi = {A_2}^2(1) |$

$\therefore A = \sqrt{{A_1}^2 + {A_2}^2 + 2A_1A_2 \cos \phi}$ ..... (6)

This is resultant amplitude.

Special Cases:

Case-I : When $\phi = 0$

i.e. the wave is in phase then

$\cos \phi = \cos 0 = 1$

Equation (6) becomes,

$A = \sqrt{{A_1}^2 + {A_2}^2 + 2A_1A_2(1)}$

$A = \sqrt{(A_1 + A_2)^2}$

$A = A_1 + A_2$

$\therefore$ *Resultant amplitude is maximum when* $\phi = 0$

Case-II : When $\phi = \pi$

i.e. the wave is in out of phase

$\cos\pi = -1$

Equation (6) becomes,

$A = \sqrt{{A_1}^2 + {A_2}^2 + 2A_1A_2(-1)}$

$A = \sqrt{(A_1 - A_2)^2}$

$A = A_1 - A_2$

$\therefore$ *Resultant amplitude is minimum when* $\phi = \pi$

Note : The intensities of the waves are proportional to the square of their amplitudes

i.e. $I \propto A^2$

$I_{max} \propto (A_{max})^2$

$I_{min} \propto (A_{min})^2$

$\frac{I_{max}}{I_{min}} = \frac{(A_1+A_2)^2}{(A_1-A_2)^2}$

Principle of superposition of waves :

When two or more waves travelling through a medium arrive at a point of the medium simultaneously, each wave produces its own displacement at that point independent of the other. Hence, the resultant displacement at that point is equal to the vector sum of the displacement due to all the waves.

Equation Of Simple Harmonic Progressive Wave:

Consider a S.H.P wave, which is travelling in positive x direction. As the wave is travelling in positive x direction when the displacement of particle at the origin is given by,

$Y = a\sin\omega t$

Now, let us consider any point P on the wave which is at a distance x from the origin. It is lags behind in the phase than that of its previous particles. Let the phase difference be δ . So, the displacement of particle at point P is given by,

$Y = a\sin(\omega t - \delta)$

But the phase difference δ (delta) is depends upon the distance x . So, the phase difference is given by, $\frac{2\pi x}{\lambda}$ in the equation.

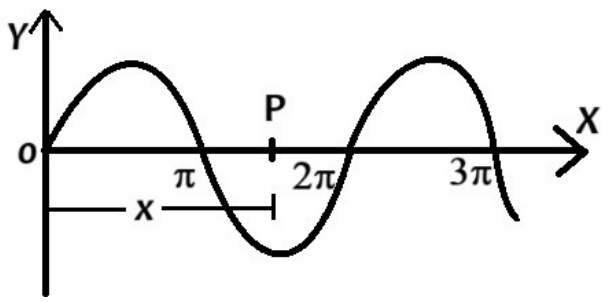
$Y = a\sin(\omega t - \frac{2\pi x}{\lambda})$

$Y = a\sin(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda})$ _____$(\because \omega = \frac{2\pi}{T})$

$\boxed{Y = a\sin 2\pi(\frac{t}{T} - \frac{x}{\lambda})}$

$\boxed{Y = a\sin 2\pi(nt - \frac{x}{\lambda})}$ _____$(\because n = \frac{1}{T})$

$Y = a\sin 2\pi n(t - \frac{x}{n\lambda})$ _____$(n\ common)$



$$Y = a \sin 2\pi n \ (t - \frac{x}{v})$$ _____$(\because v = n\lambda)$

$$Y = a \sin \frac{2\pi v}{\lambda}(t - \frac{x}{v})$$ _____$(\because n = \frac{v}{\lambda})$

$$Y = a \sin \frac{2\pi}{\lambda}(vt - x)$$

$$Y = a \sin K(vt - x)$$

Where, K ⟶ Propagation constant.

Beats :-

[***Ques. What are beats ? Give the condition for formation of beats state it's application.***]

When two sound waves of equal amplitude & slightly different frequencies travels along a same straight in a same direction, due to interference the resultant intensity at that point in a medium consist of alternate maxima & minima of sound. The maximum of sound is known as waxing & minimum of sound is known as waning. This phenomenon of alternate & periodic formation of waxing & waning of wound is known as beats.

one waxing & one waning forms one beat.

Conditions for beats :-

i. The two sound waves should have equal amplitude.
ii. The difference in frequencies of the two sound waves should be small (less than 15 Hz).
iii. The two sound waves should travel along a same straight line & in same direction.
iv. The two sound waves producing beats should be of same time.

Applications of beats :-

i. To tune the musical instruments.
ii. To detect the harmful gases in mines.
iii. To determine the unknown frequency tuning fork.
iv. For the production of wound waves of very low frequency.

Period of beat (T) :-

Define : The time interval between two successive waxing or waning known as period of beat.

Frequency of beat (n) :-

Define :- The number of beats heard per second is known as frequency of beat.

Expression for beat frequency using analytical method :-

[**Ques. What are beats show that the be frequency is equal to the different between frequencies of two sound waves.**]

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Consider two sound waves produce beats, having same amplitude a & slightly difference frequencies n & n These two waves arrive at point a medium simultaneously in phase displacement due to each wave is ,

$Y_1 = a \sin 2\pi\ n_1\ t$

$Y_2 = a \sin 2\pi\ n_2\ t$

The resultant displacement is,

$Y = Y_1 + Y_2$

$Y = a \sin 2\pi\ n_1\ t + a \sin 2\pi\ n_2\ t$

$Y = a\ (\ \sin 2\pi\ n_1\ t + \sin 2\pi\ n_2\ t\ )$

Using the trigonometric relation,

$\sin A + \sin B\ = 2 \sin \left(\frac{A+B}{2}\right) . \cos \left(\frac{A-B}{2}\right).$

$Y = 2a \sin \left[2\pi \left(\frac{n_1 + n_2}{2}\right) t\right] . \cos \left[2\pi \left(\frac{n_1 - n_2}{2}\right) t\right]$

$Y = 2a \cos \left[2\pi \left(\frac{n_1 - n_2}{2}\right) t\right] . \sin \left[2\pi \left(\frac{n_1 + n_2}{2}\right) t\right]$

$Y = a \sin 2\pi \left(\frac{n_1+n_2}{2}\right)\ t$ ..... (1) | $\because 2 \cos\theta\ . \sin\theta = \sin 2\theta$ |

Where,

$A = 2a \cos 2\pi \left(\frac{n_1-n_2}{2}\right)\ t$ ..... (2)

From equation (1) it is clear that the resultant motion is also S.H.M. with resultant amplitude A & resultant frequency $\frac{n_1+n_2}{2}$

1) **For Waxing :-**

From equation (2)

$2a \cos \left[2\pi \left(\frac{n_1 - n_2}{2}\right) t\right] = maximum$

$2a \cos \left[2\pi \left(\frac{n_1 - n_2}{2}\right) t\right] = \pm 2a$

$\cos \left[2\pi \left(\frac{n_1 - n_2}{2}\right) t\right] = \pm 1$

$2n \left(\frac{n_1 - n_2}{2}\right) t = 0, \pi, 2\pi, 3\pi$ ....

$t = 0, \frac{1}{n_1 - n_2}, \frac{2}{n_1 - n_2}, \frac{3}{n_1 - n_2}$, ...

Period of beat (T) = Time interval between two successive waxing.

$T = \frac{1}{n_1 - n_2} - 0$

$T = \frac{1}{n_1 - n_2}$

$Bead\ frequency\ (\ n\ ) = \frac{1}{T}$

$n = \frac{1}{n_1 - n_2} - 0$

$n = \frac{1}{n_1 - n_2}$

$n = n_1 - n_2$ ..... (3)

2) **For Wannings :-**

From equation (2)

$2a \cos \left[2\pi \left(\frac{n_1 - n_2}{2}\right) t\right] = minimum$

$2a \cos \left[2\pi \left(\frac{n_1 - n_2}{2}\right) t\right] = 0$

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$$\cos\left[2\pi\left(\frac{n_1 - n_2}{2}\right)t\right] = 0$$

$$2n\left(\frac{n_1 - n_2}{2}\right)t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots.$$

$$t = 0, \frac{1}{2(n_1 - n_2)}, \frac{3}{2(n_1 - n_2)}, \frac{5}{2(n_1 - n_2)}, \ldots.$$

Period of beat (T) = Time interval between two successive wannings.

$$T = \frac{3}{2(n_1 - n_2)} - \frac{1}{2(n_1 - n_2)} = \frac{2}{2(n_1 - n_2)}$$

$$T = \frac{1}{n_1 - n_2}$$

$$Bead\ frequency\ (n) = \frac{1}{T}$$

$$n = \frac{1}{\frac{1}{n_1 - n_2}}$$

$$n = n_1 - n_2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Stationary Waves:

Define : When two waves having same amplitude, same frequency and same wavelength travels along same straight line but in opposite direction. They produce interference then resultant wave do not move either in forward or in backward direction but remains stationary. Such waves are known as " Standing or stationary waves".

Stationary waves are of two types:

1. Transverse stationary wave and
2. Longitudinal stationary wave.

Experimentally, stationary wave is produced from a single progressive wave. The progressive wave is made up of incident on some rigid support and reflected wave is obtained. The incident wave and reflected wave together produce a stationary wave.

❖ **Characteristics Of Stationary Wave :-**

1. Stationary waves are formed due to interference between two exactly identical waves travelling along and same straight in a medium but in opposite direction.
2. In stationary waves there are certain equidistance point, at which amplitude is zero, which are called "Nodes".
3. In stationary wave there are certain points at which the amplitude is maximum are called "Antinodes".
4. The distance between two successive nodes or antinodes is $\frac{\lambda}{2}$.
5. Nodes and antinodes are alternately situated and equispaced. The distance between any node and adjacent antinodes is $\frac{\lambda}{4}$.
6. The part (not distance) of wave between two successive nodes, is called 'loop'. All the particles in one loop are in same phase, while the particles in successive loop are out of phase.
7. The stationary wave does not transfer sound energy through the medium.
8. The stationary waves are periodic in time as well as periodic in space.

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Distinguish between Progressive wave and Stationary waves :-

Sr. No.	Progressive Wave	Stationary Wave
1	These waves are produced when a disturbance is created in medium.	These waves are produced due to interference between two exactly identical waves travelling in opposite direction.
2	These waves are propagated only in forward direction.	These waves are not propagated in any direction.
3	In such wave sound energy is transferred through the medium.	In such wave energy do not transfer.
4	All the particles vibrate with same amplitude.	All the particles vibrate with different amplitude.
5	Each particle of medium vibrates.	The particles at node are permanently at rest.
6	In this wave nodes, antinodes and loop are not formed.	In this wave nodes, antinodes and loop are formed.

Modes of vibration on stretched string :-

[Explain different modes of vibrating string. Obtain an expression for frequencies in each case ? Or
Explain how stationary waves are formed along a string. Calculate the corresponding frequencies in each case ?]

Consider a uniform string of length l stretched between two rigid supports. When the string is pulled perpendicular to its length and released. Then transverse waves are travels along the string. The wave gets reflected from rigid support and the stationary waves are formed. When a steady pattern of stationary wave is formed, then let P be the number of loops formed. Therefore, the length of the string between two extreme nodes is,

$l = P \times \frac{\lambda}{2}$

$\therefore \lambda = \frac{2\,l}{P}$ ..... (1)

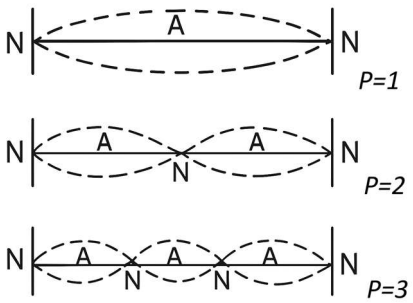
The frequency of vibrating string is,

$n = \frac{v}{\lambda}$ $\quad\quad\quad\quad$ $\because v = n\lambda$

But $v = \sqrt{\frac{T}{m}}$

Equation (1) becomes,

$n = \frac{P}{2\,l}\sqrt{\frac{T}{m}}$



Thus, the string can vibrate with different-frequencies, called modes of vibration.

Fundamental frequencies Ist harmonic,

$l = \frac{\lambda}{2}$

IInd Harmonic, Ist overtone,

$l = \frac{2\lambda}{2} = \lambda$

IIIrd Harmonic, Ist overtone,

$l = \frac{3\lambda}{2}$

When the string vibrates forming one loop (P=1), Then the modes of vibration is fundamental OR

I[st] Harmonic and the frequency of string is,

$n_1 = \frac{1}{2\,l}\sqrt{\frac{T}{m}}$ ..... (2)

This is the lowest frequency with which the string can vibrates.

When the string can vibrate forming two loops (P=2), then the mode of vibration is called II[nd] Harmonic or I[st] overtone.

The frequency of string is,

$n_2 = \frac{2}{2\,l}\sqrt{\frac{T}{m}}$

$n_2 = 2n_1$ $\because From\ Equation\ (1)\ \&\ (2)$

When the string can vibrates forming three loop (P = 3), then the mode of vibration is called as III[rd] Harmonic OR II[nd] overtone. Therefore, the frequency of string is,

$n_3 = \frac{3}{2\,l}\sqrt{\frac{T}{m}}$

$n_3 = 3n_1$

$n_1 : n_2 : n_3 = 1 : 2 : 3$

The string can vibrate with the fundamental frequency and the frequencies which are integral multiple of fundamental frequencies.

Distinguish between Harmonic and overtone :-

Sr. No.	Harmonic	Overtone
1	Integral multiple of fundamental frequency is called harmonic.	Vibrations at higher frequencies which are actually, present in addition to the fundamental frequency is called overtone.
2	Fundamental frequency itself is the I[st] harmonic.	Fundamental frequency is not I[st] overtone.
3	In some vibrations harmonic may absent.	All the overtones are always present.

Laws of vibrating string -

We have the fundamental frequency of vibrating string is,

$n = \frac{1}{2\,l}\sqrt{\frac{T}{m}}$

From this relation, we have following laws :

1. **Law of length :-**
   The fundamental frequency of vibrating string is inversely proportional to the length of string, when the tension and mass per unit length of string are constant.
   $i.e, \quad n \propto \frac{1}{l} \quad \because T\ \&\ m\ are\ constant$
2. **Law of tension :-**
   The fundamental frequency of vibrating string is directly proportional to the square root of tension applied, when length and linear density of string are constant.
   $i.e, \quad n \propto \sqrt{T} \quad \because l\ \&\ m\ are\ constant$
3. **Laws of mass :-**
   The fundamental frequency of vibrating string is inversely proportional to the square root of

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linear density of string, when length and tension applied are constant.

$i.e, \quad n \propto \frac{1}{\sqrt{m}} \qquad \because l \ \& T \ are \ constant$

Dependence of fundamental frequency on radius and density of string :-

The fundamental frequency of vibrating string is,

$n = \frac{1}{2\,l}\sqrt{\frac{T}{m}}$ ..... (1)

Let,

r = radius of string

$\varrho = density \ of \ string$

Mass per unit length is,

$m = \pi r^2 \varrho \qquad \because m = v \times \varrho$

$\because \frac{M}{l} = \frac{\pi r^2 l}{l}$

$\because m = \pi r^2$

Putting this value in equation (1)

$n = \frac{1}{2\,l}\sqrt{\frac{T}{\pi r^2 \varrho}}$

$n = \frac{1}{2\,r\,l}\sqrt{\frac{T}{\pi \varrho}} \qquad \because \sqrt{r^2} = r$

So, , $n \propto \frac{1}{r}$, when l ,T and ϱ are constant. This is the law of radius and $n \propto \frac{1}{\sqrt{\varrho}}$, when l ,r and T are constant

This is the law of density.

Free vibrations :- (Undamped Vibrations)

Define - When a body is displaced from its mean position and released, it begins to vibrates about its mean position. Such vibration is called as free vibrations.

Or

Define - The natural vibrations of body with constant amplitude and frequency are known as free vibrations.

The frequency with which the body vibrates is known as the natural frequency of body. The natural frequency of body depends upon elastic property, dimensions (length, mass) etc. of the body.

[Amplitude of natural vibrations gradually decreases and body comes to rest after some times. Such a vibrations with decreasing amplitude are called damped vibrations].

Ex - Simple pendulum.

Forced Vibrations :-

Define - When an external periodic force is applied to a body, the body begins to vibrate with the frequency equal to frequency of external force. If the frequency of external force is different from natural frequency of body, then such vibrations are called forced vibrations.

In such vibrations amplitude is very small. Amplitude of force vibration is found to be increased with,

1. Increase in magnitude of a periodic force.
2. Decrease in difference between the frequency of force and natural frequency of body.

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Ex - 1)When a vibrating tuning fork is held with its stem in contact with a table top, then top begins to perform forced vibrations.

2) When the soldiers are marching on a suspended bridge then the vibrations produced in the bridge are the forced vibrations.

Resonance :-

Define - The amplitude of forced vibration is maximum, when the frequency of applied force is same as natural frequency of body. Such vibrations are called as resonant. Vibration and the phenomenon are known as resonance.

In resonance when the body vibrates in unison with the external force, then amplitude of vibration becomes maximum.

ex - 1) When we tune the radio - receiver for the incoming signal. We adjust the maximum frequency of oscillation in the receiver equal to that of the incoming signal.

2) When the speed of vehicle is increasing beyond certain limit the different parts of vehicle vibrates due to resonance and large noise is produced.

Disadvantage Of Resonance :-

1. It the frequency of clapping of hands by a cheering audience is same as the natural frequency of the roof, the roof may be fall down due to resonance.
2. When speed of an Aircraft increases, different parts are forced to vibrate. Resonance is underivable.

Vibration of air column :-

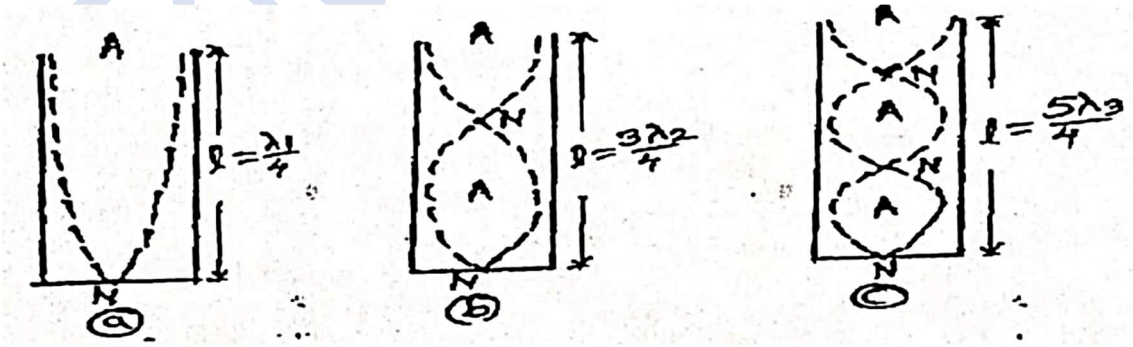
1) Pipe closed at one end :- (Close organ pipe)

Ques :- Show that only odd harmonics are present in the vibration of an air column, in a pipe close at one end (neglecting the end correction) OR

Describe first three modes of vibration of an air column enclosed by a pipe close at one end and determine the corresponding frequency.

When a source of sound (tuning fork) is held near the open end of the pipe then the longitudinal wave travels through an air column inside the pipe. These waves get deflected from the close end of pipe forming a node. Then the stationary waves are formed. The antinodes situated at the open end of pipe, because the particles at open end are free to vibrate.

First three nodes of vibrations are shown in figures above –



1. If the mode of vibration is as shown in fig (a) then length of air column (l) = $\frac{\lambda_1}{4}$

   $\therefore \lambda_1 = 4l$

   Frequency of air column in $n_1 = \frac{v}{\lambda_1}$

$\therefore\ n_1 = \frac{v}{4l}$ ……(1) $(\because v = n\lambda)$

This is the fundamental frequency or Ist Harmonic for a pipe close at one end.

2. If the mode of vibration is as shown in figure (b). In this case the air column vibrates forming two nodes and two antinodes.

Length of air column = $l = \frac{3\lambda_2}{4} = \lambda_2$

$\therefore \lambda_2 = \frac{4\,l}{3}$

Frequency of air column is,

$n_2 = \frac{v}{\lambda_2} = \frac{v}{4\,l/3}$

$n_2 = \frac{3\,v}{4\,l}$

$\therefore\ n_2 = 3n_1$ …… (2) From (1)

3. If the mode of vibration is as shown in fig. (c). In this air column air vibrates forming 3 nodes and 3 antinodes.

Length of air column = $l = \frac{5\lambda_3}{4}$

$\therefore \lambda_3 = \frac{4\,l}{5}$

Frequency of air column is,

$n_3 = \frac{v}{\lambda_3} = \frac{v}{4\,l/5}$

$n_3 = \frac{5\,v}{4\,l}$

$\therefore\ n_3 = 5n_1$ …… (3) From (2)

Where n_3 is called Vth harmonic or second overtone.

Thus, from equation (1), (2) and (3)

$n_1 : n_2 : n_3 = 1 : 3 : 5$

Therefore, in case of an air column in a pipe close at one end only odd harmonics are present.

Formula :-

Harmonic= (2P+1) , Where P is overtone

Overtone= $\frac{f-1}{2}$, where f is harmonic

Pipe open at both end (Open organ pipe) :-

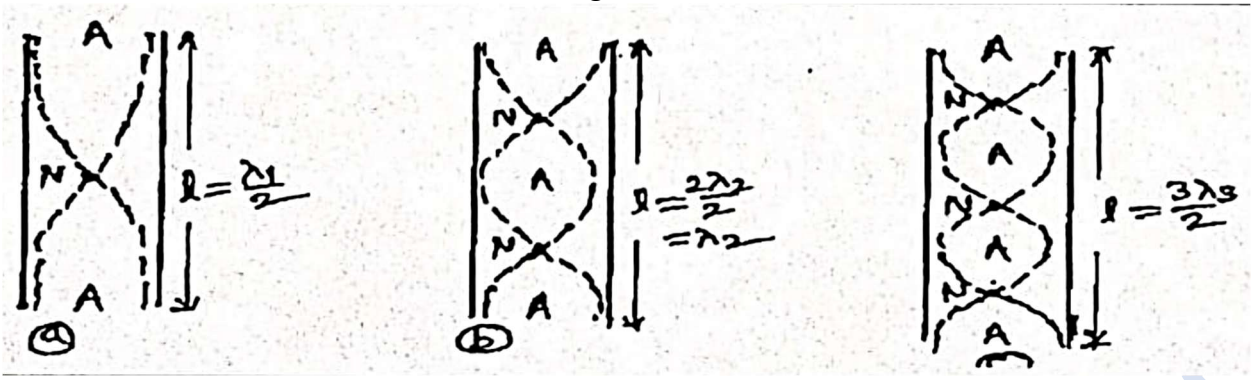
Ques :- Describe Ist modes of vibration of an air column enclosed by a pipe open at both ends. Hence determine the corresponding frequencies. OR

Show that all harmonic is present in the vibration of an air column enclosed by a pipe open at both ends. (Neglecting end correction).

When a source of sound (tuning fork) is held near the open end of pipe, the longitudinal waves travel through an air column inside the pipe. These waves get reflected and forms a stationary wave in the pipe. In this case the antinodes are situated at both the ends because the particles at open end are free to vibrates.

EducationalNotes

First three modes of vibration are shown in fig.



1. If the modes of vibration are as shown in fig. (A) then,

   Length of air column = $l = \frac{\lambda_1}{2}$

   $$\lambda_1 = 2\,l$$

   Frequency of air column,

   $$n_1 = \frac{v}{\lambda_1}$$

   $$n_1 = \frac{v}{2\,l} \quad \text{..... (1)}$$

   This is the fundamental frequency or 1st harmonic in case of pipe open at both ends.
2. If the mode of vibration is as shown in figure (B), in this case the air column is made to vibrate forming two nodes and three antinodes.

   Length of air column = $l = \frac{2\lambda_2}{2} = \lambda_2$

   $$\lambda_2 = l$$

   Frequency of air column,

   $$n_2 = \frac{v}{l}$$

   $$n_2 = 2n_1 \quad \text{..... (2)} \qquad \text{From equation (1)}$$

   Where n_2 is called II[nd] harmonic or I[st] overtone.
3. Now consider the third mode of vibration in which the air column is made to vibrate forming 3 nodes and 4 antinodes.

   Length of air column = $l = \frac{3\lambda_3}{2}$

   $$\lambda_3 = \frac{2\,l}{3}$$

   Frequency of air column,

   $$n_3 = \frac{v}{\lambda_3}$$

   $$n_3 = \frac{3v}{2\,l}$$

   $$n_3 = 3n_1 \quad \text{..... (3)} \qquad \text{From equation (1)}$$

   Where n_3 is called III[rd] harmonic or II[nd] overtone.

   Thus, from equation (1), (2) and (3)

   $$n_1 : n_2 : n_3 = 1 : 2 : 3$$

   Therefore, in case of air column vibrating in pipe open at both ends all harmonic are present.

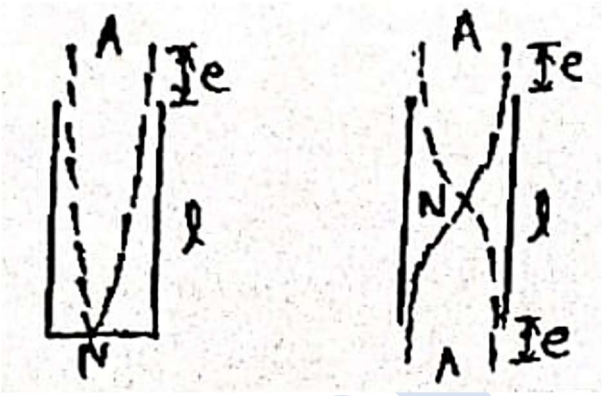
Formula :-

Harmonic= (P+1) , Where P is overtone

Overtone= $f - 1$, Where f is harmonic

Educational Notes

End correction (e) :-



Define : - Air is free to move at the open end of the pipe. Therefore, in case of open pipe the antinode is formed slightly outside the open end. This distance from the open end is called "end correction" .
It is denoted by 'e'.
For a pipe of circular cross section the end correction is, e = 0.3d
Where d - inner diameter of tube / pipe with the end correction, the fundamental frequency in a pipe close at one end is,

$n = \frac{v}{4\,(l+e)}$

for open pipe,

$n = \frac{v}{2\,(l+2e)}$

For pipe closed at one end, fundamental frequency is,

$n = \frac{v}{4l}$

$v = 4nl$

$v = 4n\,(l+e)$ ..... by considering end correction

Hence by knowing $n, l \; and \; e$ velocity of sound in air at room temperature can be determined.

For a pipe closed at one end :-

$v = 4nl$

$4n_1l_1 = 4n_2l_2$

$4\,n_1\,(l_1+e) = 4\,n_2\,(l_2+e)$

$n_1l_1 + n_1e = n_2l_2 + n_2e$

$n_1l_1 - n_2l_2 = e\,(n_2-n_1)$

$e = \frac{n_1l_1-n_2l_2}{n_2-n_1} \quad or \quad e = \frac{n_2l_2-n_1l_1}{n_1-n_2}$

For a pipe closed at one end :-

$v = 2nl$

$2n_1l_1 = 2n_2l_2$

$n_1\,(l_1+2e) = n_2\,(l_2+2e)$

$n_1l_1 + 2\,n_1e = n_2l_2 + 2\,n_2e$

$n_1l_1 - n_2l_2 = 2\,e\,(n_2-n_1)$

$e = \frac{n_1l_1-n_2l_2}{2\,(n_2-n_1)} \quad or \quad e = \frac{n_2l_2-n_1l_1}{2\,(n_1-n_2)}$

Limitation of end correction :-

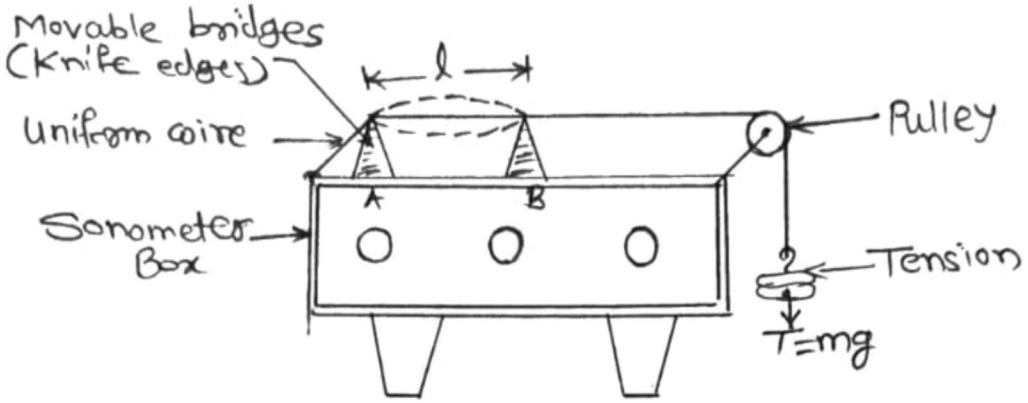
1. Inner diameter of tube must be uniform throughout its length.
2. Effect of flow of air outside tube is to be neglected.
3. Effect of temperature of air outside is to be neglected.
4. Tip of prong of vibrating tuning fork must be held horizontally at the centre and at a small distance above open end of the tube.

Educational Notes

Sonometer :

[***Ques*: *Describe the construction working of sonometer ? or Describe the construction of sonometer. Explain its use to determine the frequency of tuning fork ?***]

Construction :-



Sonometer consist of hollow, rectangular wooden box, provided with two or three holes to communicate the air form inside to the atmospheric air. A thin & uniform wire is fixed at one end of the box and it passes over the two movable bridges A & B. Then it passes over a pulley at another end of the box. The tension can be applied to the free and of the wire. The length of wire between the bridges can be increased by moving one of the bridges.

Procedure (use of sonometer) –

Paper rider method : Initially tension of suitable value is applied to the free end of sonometer wire. The length of wire between the bridges is adjusted minimum. A small piece of paper called rider is placed at the centre of length of wire between the bridges. A tuning fork of unknown frequency is vibrated & it is brought in contact with sonometer box. The length of wire between the bridges is gradually increased, keeping the rider always at the centre. As the length of wire changes, its natural frequency also changes when its natural frequency becomes equal to frequency of fork, resonance takes place & the wire vibrates with maximum amplitude. Therefore, the paper rider is thrown off or flies off.

The length of wire between the bridges is measured (l) , called resonating length. Mass (M) & length (l) of sonometer wire is measured from which liner density (m) of wire is determined . Therefore, the frequency of tuning fork is determined by the formula,

$$n = \frac{1}{2\,l}\sqrt{\frac{T}{m}}$$

Verification of Laws of Vibrating String by Sonometer :-

[$Question : State\ any\ two\ laws\ of\ vibrating\ string\ varify\ (1)\ law\ of\ length\ , (2) law\ of\ tension\ (3) law\ of\ mass$]

1) **Verification of law of length :-**

[Draw diagram of Sonometer]

It states that,

$n \propto \frac{1}{l}$, when T & m are constant

i.e. $n\,l = constant$

Educational Notes

A constant tension T is applied to free end of sonometer wire & the resonating lengths for the tuning forks of known frequencies n_1, n_2, n_3 ..... are determined. Suppose these are l_1, l_2, l_3 ..... respectively.

As the same wire is used throughout the experiment, therefore the linear density of wire remains constant.

$\therefore It\ is\ found\ that,$

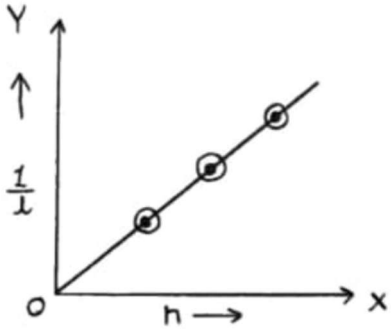
$n_1 l_1 = n_2 l_2 = \ldots\ldots\ldots = constant$

$i.e, \quad nl = constant$

$n \propto \frac{1}{l}$ .....$\because$ T & m are constant

Thus, the law of length is verified.

If the graph between (n) against ($\frac{1}{l}$) is plotted, it comes out as a straight line as shown below.



2) Verification of law of tension :-

[Draw diagram of Sonometer]

It states that,

$n \propto \sqrt{T} \quad \because \ when\ l\ \&\ m\ are\ constant$

$i.e, \quad \frac{n}{\sqrt{T}} = constant$

Initially length of wire between the bridges is kept constant. A set of tuning forks of known frequencies n_1, n_2, n_3 ..... is taken. The tension in sonometer wire is adjusted so that the paper rider will be flies off (resonance accrued) . Let T_1, T_2, T_3 ..... Be the tension for the tuning forks of frequencies n_1, n_2, n_3 ..... respectively.

As the same wire is used through at the experiment, linear density of wire remains constant.

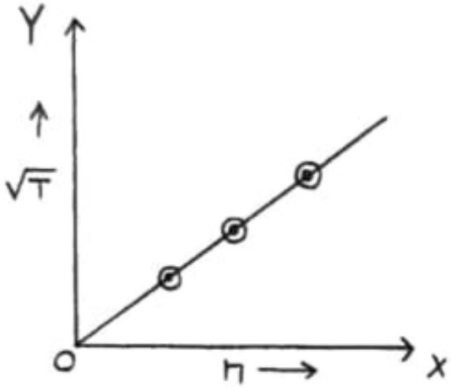
It is found that,

$\frac{n_1}{\sqrt{T_1}} = \frac{n_2}{\sqrt{T_2}} = \ldots\ldots\ldots\ldots = constant$

$i.e, \quad \frac{n}{\sqrt{T}} = constant$

$n \propto \sqrt{T} \quad \because \ l\ \&\ m\ are\ constant$

If the graph between n and $\sqrt{T}$ is plotted, it comes out as a straight line as shown in figure.



3) Verification Of Law of Mass :

[Draw diagram of Sonometer]

This law can not be verified directly as the frequency & linear density cannot be varied continuously. Consider different sonometer wires of mass per unit lengths m_1,m_2..... . Then by using tuning fork of known frequency n & keeping the tension constant, the corresponding responding lengths l_1, l_2..... are determined respectively.

It is found that,

$l_1\sqrt{m_1} = l_2\sqrt{m_2}$

$l\sqrt{m} = constant$

But by first law of a vibrating string, $n \propto \frac{1}{l}$

Therefore, we get that,, $n \propto \frac{1}{\sqrt{m}}$, if T and l are constant. This is the third law of vibrating string.
In this way, laws of a vibrating string are verified by using a sonometer.

Musical Instruments:

The musical Instrument can be classified into four types.

1. Stringed Instruments
2. Wind Instruments
3. Percussion Instruments
4. Solid Instruments

1) Stringed Instruments :
In these stretched strings are made to vibrate by
a. Striking with a hammer
b. Plucking with the finger
c. Bowing with a bow
Ex. (a)Pianoforte , (b)Sitar , Guitar , Veena , (c)Violin , Santor

2) Wind Instruments
In these instruments an air column of varying length is made to vibrate by blowing a jet of air at one end.
Ex. Flute, Bugle, Bassoon, Harmesnium etc

3) Percussion Instruments
The instruments in which a circular stretched membrane is made to vibrate up and down over a hollow chamber of air by striking with palm and fingers.
Ex. Drums, Tabala etc.

Equation of Stationary Wave :

Ques : Explain how stationary waves are formed analytically ?
or
Give the analytical treatment of stationary wave. Prove that the nodes and antinodes are alternately situated and equispaced

Consider two S.H.P. waves of equal amplitude a , frequency n & wavelength λ , travelling along same line & in opposite direction.
The equation of wave travelling in negative direction of X-axis is ,

$$Y_1 = a \sin 2\pi \left(nt + \frac{x}{\lambda}\right)$$

& The equation of wave travelling in positive direction of X-axis is,

$$Y_2 = a \sin 2\pi \left(nt - \frac{x}{\lambda}\right)$$

$\therefore$ The resultant displacement is given by,

$$Y = Y_1 + Y_2$$

$$Y = a \sin 2\pi \left(nt + \frac{x}{\lambda}\right) + a \sin 2\pi \left(nt - \frac{x}{\lambda}\right)$$

$$Y = a \left[\sin 2\pi \left(nt + \frac{x}{\lambda}\right) + \sin 2\pi \left(nt - \frac{x}{\lambda}\right)\right]$$

Using the Trigonometric relation,

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) . \cos\left(\frac{A-B}{2}\right)$$

Educational Notes

$$\therefore Y = 2a \sin\left[2\pi\left(\frac{nt + \frac{x}{\lambda} + nt - \frac{x}{\lambda}}{2}\right)\right] . \cos\left[2\pi\left(\frac{nt + \frac{x}{\lambda} - nt + \frac{x}{\lambda}}{2}\right)\right]$$

$$\therefore Y = 2a \sin\left[2\pi\left(\frac{2nt}{2}\right)\right] . \cos\left[2\pi\left(\frac{2x}{2\lambda}\right)\right]$$

$$\therefore Y = 2a \cos\left(\frac{2\pi x}{\lambda}\right) . \sin(2\pi nt)$$

$\therefore Y = A \sin 2\pi nt$..... (1)

Where,

$A = 2a \cos\left(\frac{2\pi x}{\lambda}\right)$..... (2)

Equation (1) shows that, the equation of stationary wave, because of absence of term 'x' . Hence it is a stationary wave. Here A is the amplitude of resultant stationary wave.

- Condition for Antinode :
  The point of medium at which the amplitude is maximum is called antinode.
  From equation (2)
  $\therefore 2a \cos\left(\frac{2\pi x}{\lambda}\right) = maximum\ (\pm 2a)$
  $\therefore \cos\left(\frac{2\pi x}{\lambda}\right) = \pm 1$
  $\therefore \frac{2\pi x}{\lambda} = 0, \pi, 2\pi, 3\pi$.....
  $\therefore x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$ .....
  Distance between two successive antinodes is = $\frac{\lambda}{2}$ = 0
  $= \frac{\lambda}{2}$

- Condition for Node :
  The point of medium at which the amplitude is minimum is called Node.
  $\therefore From\ equation\ (2)$
  $\therefore 2a \cos\left(\frac{2\pi x}{\lambda}\right) = minimum$
  $\therefore \cos\left(\frac{2\pi x}{\lambda}\right) = 0$
  $\therefore \frac{2\pi x}{\lambda} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$ .....
  $\therefore x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$ .....
  Distance between two Successive nodes is = $\frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2}$
  Now the distance between Node & adjacent Antinode is,
  $= \frac{\lambda}{4} = 0$
  $= \frac{\lambda}{4}$
  Thus, in stationary wave Nodes & Antinodes are alternately situated and equispaced.

Educational Notes